Preconditioned iterative solvers for immersed finite element methods

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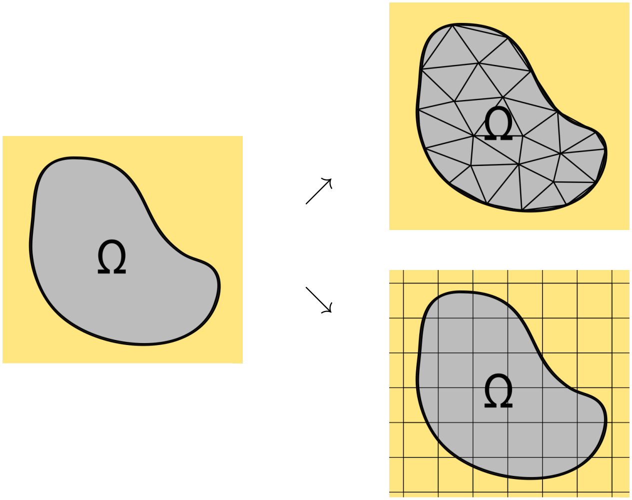
May 19, 2017

Outline

1. Immersed finite element methods

2. Conditioning analysis and preconditioning

Concept

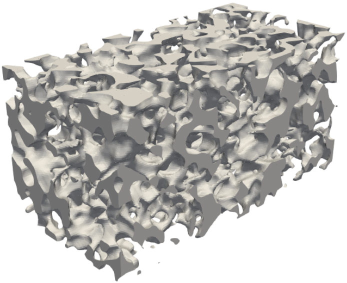


Conforming FEM

Immersed FEM

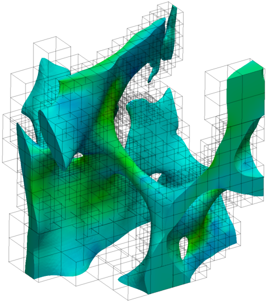
Motivation (1): complex geometries

[C.-Z. Qin]



Sinthered glass beads

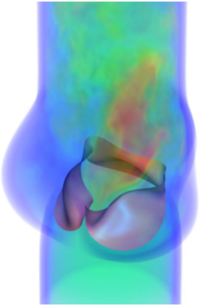
[C.V. Verhoosel 2015]



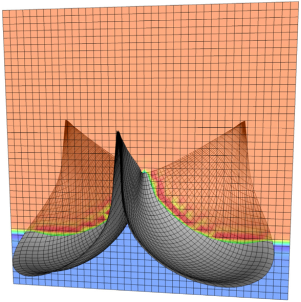
Trabecular bone

Motivation (2): time dependent domains

[M.-C. Hsu 2014]



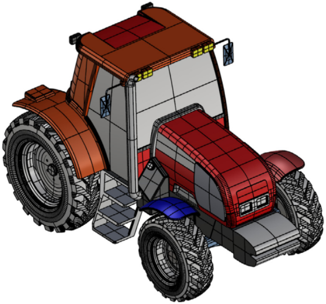
[D. Kamensky 2017]



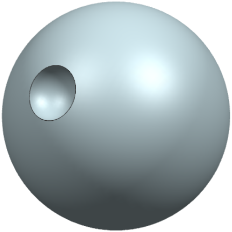
Transient simulation of prosthetic heart valve

Motivation (3): isogeometric analysis (IGA)

[M.-C. Hsu 2016]

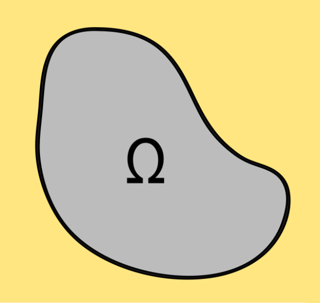


External flow around CAD-geometry



IGA on trimmed CAD-geometries

Imposing boundary conditions



Domain

$$\text{strong} \begin{cases} -\Delta u & = f \text{ in } \Omega \\ u & = g^D \text{ on } \Gamma^D \subset \partial\Omega \\ n \cdot \nabla u & = g^N \text{ on } \Gamma^N \subset \partial\Omega \end{cases}$$

$$\text{weak} \begin{cases} \text{find } w \in \mathcal{H}_0^1(\Omega) \text{ s.t. :} \\ \quad a(v, w) = b(v) - a(v, q) \\ \text{for all } v \in \mathcal{H}_0^1(\Omega) \end{cases}$$

$$a(v, u) = \int_\Omega \nabla v \cdot \nabla u \, \mathrm{d}V$$

$$b(v) = \int_\Omega v f \, \mathrm{d}V + \int_{\Gamma^N} v g^N \, \mathrm{d}S$$

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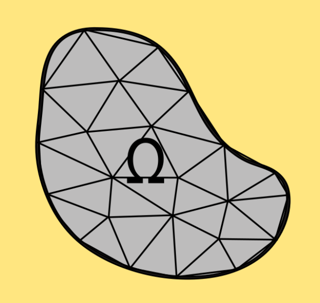

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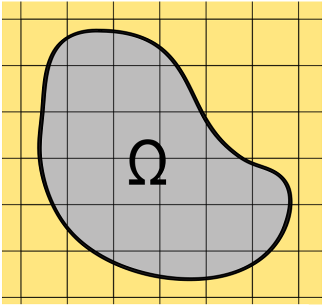

Immersed FEM

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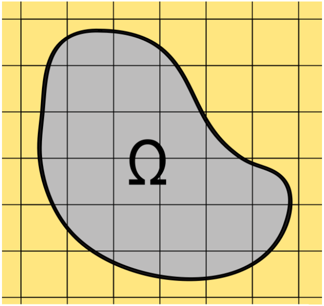

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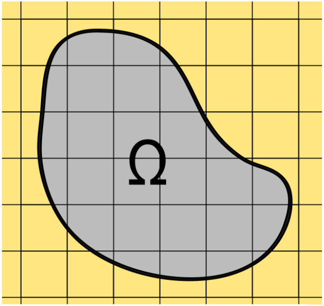

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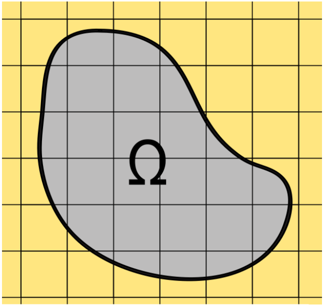

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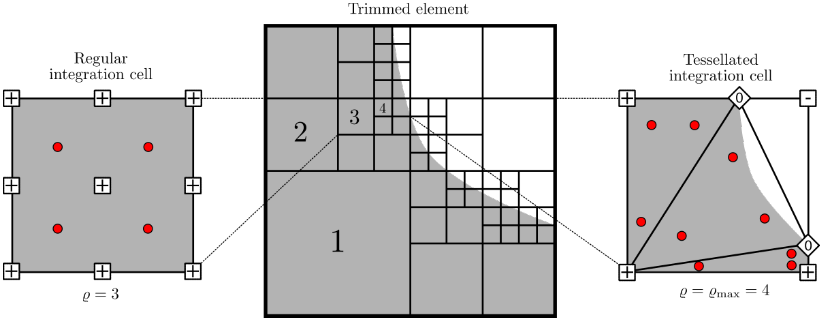
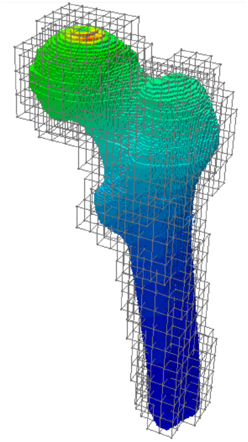
Integration of trimmed elements

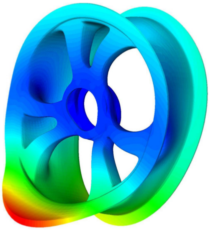


[C.V. Verhoosel 2015]

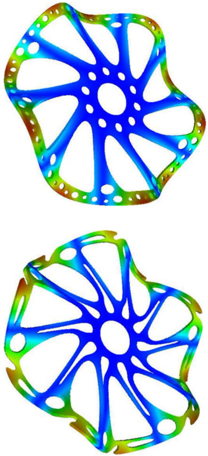
Static elasticity problems



[Ruess 2013]

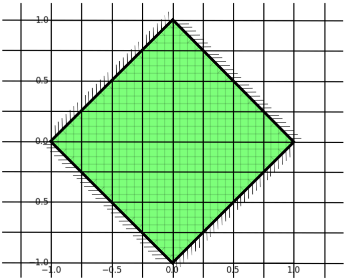


[Schillinger 2012]

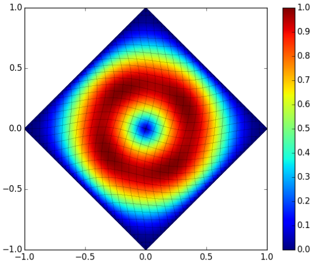


[Rank 2012]

Dynamic elasticity problems

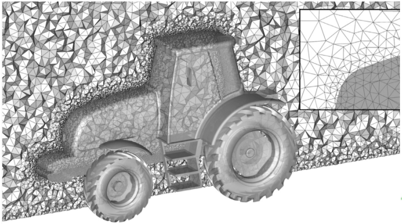


Grid



Initial condition

Large Eddy Simulations (LES)

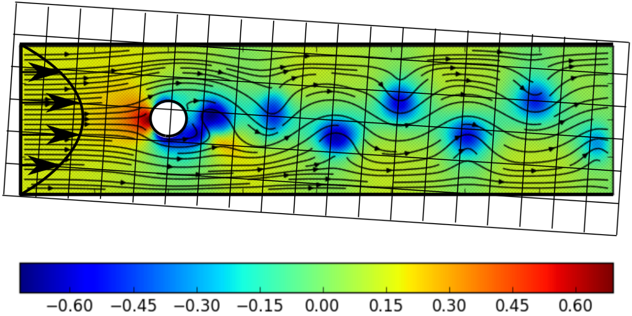


[M.-C. Hsu 2016]

[F. Xu 2016]

Variational Multiscale modeling of Navier-Stokes

Transient flow problems



Von Karman vortex street in Navier-Stokes

Outline

Conditioning of immersed methods

From weak form to linear system

function	$v^h \left(= \mathbf{\Phi}^T \mathbf{v}\right)$	$\Leftrightarrow$	$\mathbf{v}$	coefficient vector
weak form	$a(v^h, u^h) = b(v^h)$	$\Leftrightarrow$	$\mathbf{A}\mathbf{u} = \mathbf{b}$	linear system

condition number: $\kappa(\mathbf{A}) = \|\mathbf{A}\| \|\mathbf{A}^{-1}\|$

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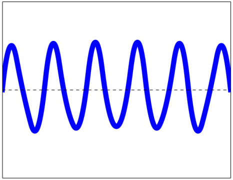

v^h: highest periodicity

$$\|\mathbf{A}\| = \max_{\mathbf{v}\neq 0} \frac{\|\mathbf{A}\mathbf{v}\|}{\|\mathbf{v}\|} = \max_{\|\mathbf{v}\|=1} \left\| a\left(\mathbf{\Phi}, v^h\right) \right\|$$

SPD systems :

$$\max_{\|\mathbf{v}\|=1} \mathbf{v}^T \mathbf{A}\mathbf{v} = \max_{\|\mathbf{v}\|=1} a(v^h, v^h)$$

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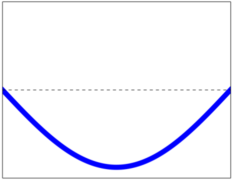

v^h: lowest periodicity

$$\|\mathbf{A}^{-1}\| = \max_{\mathbf{v}\neq 0} \frac{\|\mathbf{v}\|}{\|\mathbf{Av}\|} = \max_{\|\mathbf{v}\|=1} \frac{1}{\|a\left(\mathbf{\Phi}, v^h\right)\|}$$

SPD systems :

$$\max_{\|\mathbf{v}\|=1} \frac{1}{\mathbf{v}^T\mathbf{Av}} = \max_{\|\mathbf{v}\|=1} \frac{1}{a(v^h, v^h)}$$

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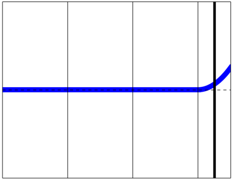

~~v^h: lowest periodicity~~

v^h: smallest volume fraction!

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$$\eta = \min_e \frac{|\Omega^e_{\text{cut}}|}{|\Omega^e_{\text{uncut}}|}$$

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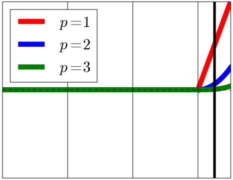

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order dependence!

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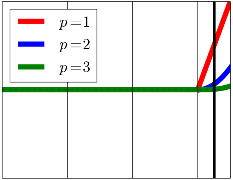

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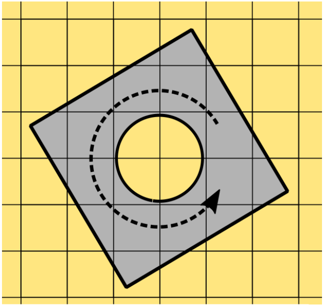
$$\kappa \propto \eta^{-(2p+n)}$$

Verification of conditioning analysis

Domain:

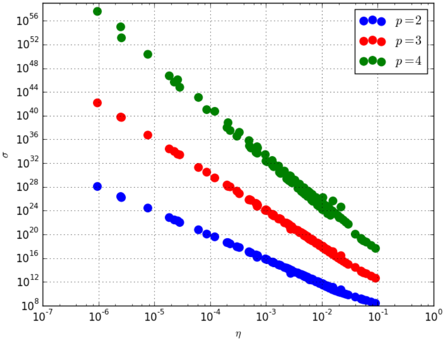


Experiment

- Domain rotated over grid
- *Different* discretizations of the *same* problem with the *same* mesh size
- κ (condition number) and η (volume fraction) at every separate rotation

Verification of conditioning analysis

Stokes:



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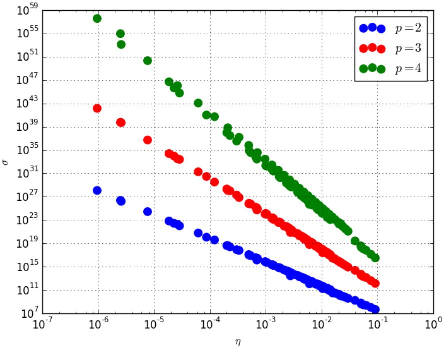
Navier-Stokes:



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Preconditioning concept

Problem analysis

Functions v^h and corresponding coefficient vectors $\mathbf{v}$ with:

$$v^h \ll \mathbf{v} \tag{1}$$

Preconditioning the space

- Replace basis $\mathbf{\Phi}$ by the manipulated basis $\bar{\mathbf{\Phi}} = \mathbf{S\Phi}$
- For nonsingular $\mathbf{S}$ the bases $\mathbf{\Phi}$ and $\bar{\mathbf{\Phi}}$ span the same space
- Choose matrix $\mathbf{S}$ such that the problem in (1) is precluded

Implementation

The preconditioned system becomes:

$$\mathbf{SAS}^T\bar{\mathbf{u}} = \mathbf{Sb}, \quad \mathbf{u} = \mathbf{S}^T\bar{\mathbf{u}}$$

This has the same eigenvalues as the left preconditioned system:

$$\mathbf{S}^T\mathbf{SAu} = \mathbf{S}^T\mathbf{Sb}$$

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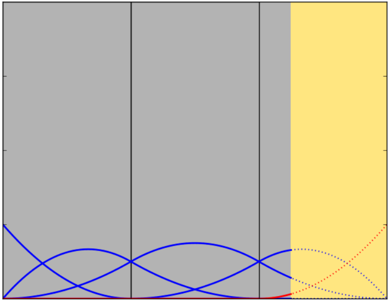

Original basis $\mathbf{\Phi}$

Small basis functions

If a basis function ϕ is small, then the (unit) vector $\|\mathbf{w}\| = 1$ corresponding to $w^h = \phi$ yields $\|\mathbf{A}\mathbf{w}\| \ll 1$

$$\|\mathbf{A}^{-1}\| = \max_{\mathbf{v}\neq 0} \frac{\|\mathbf{v}\|}{\|\mathbf{A}\mathbf{v}\|} \geq \frac{\|\mathbf{w}\|}{\|\mathbf{A}\mathbf{w}\|} \gg 1$$

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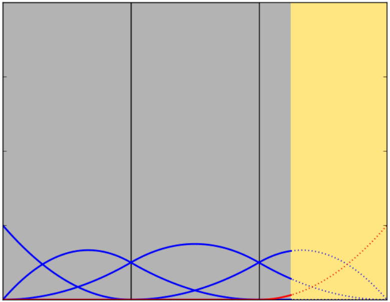

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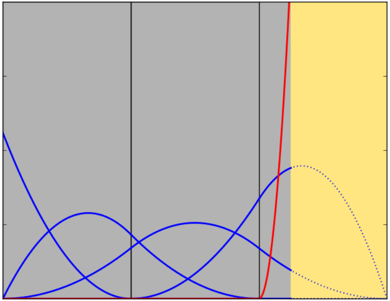

Scaled basis $\widetilde{\mathbf{\Phi}} = \mathbf{D}\mathbf{\Phi}$

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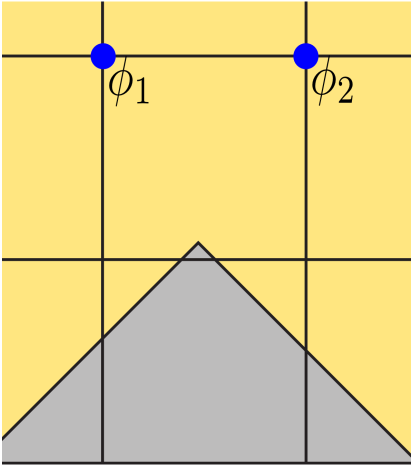
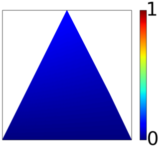
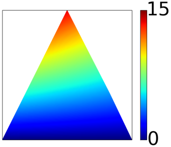
What **S** does (2): Local orthonormalization



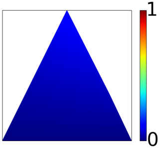
What S does (2): Local orthonormalization



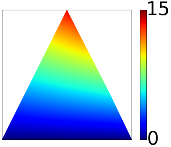
ϕ_1



$\widetilde{\phi}_1 = \phi_1 / \|\phi_1\|$



ϕ_2



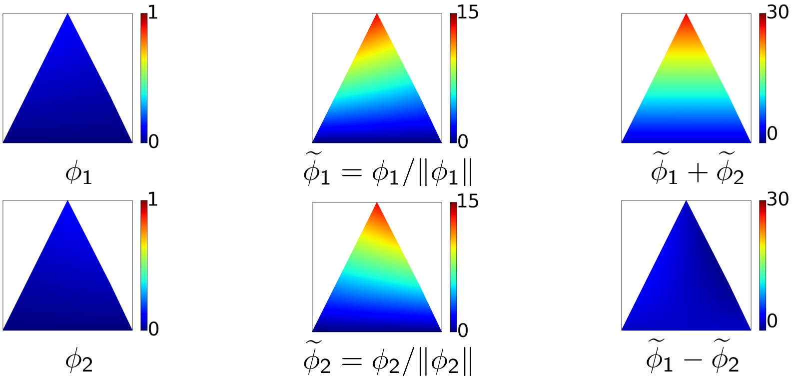
$\widetilde{\phi}_2 = \phi_2 / \|\phi_2\|$

Quasi linear dependencies

If basis functions $\widetilde{\phi}_1$ and $\widetilde{\phi}_2$ are very similar, then the vector $\|\mathbf{w}\| = \sqrt{2}$ corresponding to $w^h = \widetilde{\phi}_1 - \widetilde{\phi}_2$ yields $\|\mathbf{DADw}\| \ll 1$

$$\|\mathbf{DAD}^{-1}\| = \max_{\mathbf{v} \neq 0} \frac{\|\mathbf{v}\|}{\|\mathbf{DADv}\|} \geq \frac{\|\mathbf{w}\|}{\|\mathbf{DADw}\|} \gg 1$$

What **S** does (2): Local orthonormalization



Quasi linear dependencies

If basis functions $\widetilde{\phi}_1$ and $\widetilde{\phi}_2$ are very similar, then the vector $\|\mathbf{w}\| = \sqrt{2}$ corresponding to $w^h = \widetilde{\phi}_1 - \widetilde{\phi}_2$ yields $\|\mathbf{DADw}\| \ll 1$

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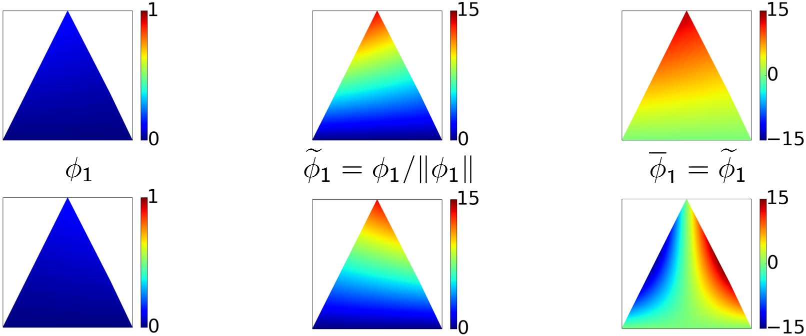
What S does (2): Local orthonormalization



ϕ_1 | $\widetilde{\phi}_1 = \phi_1 / \|\phi_1\|$ | $\overline{\phi}_1 = \widetilde{\phi}_1$

ϕ_2 | $\widetilde{\phi}_2 = \phi_2 / \|\phi_2\|$ | $\overline{\phi}_2 \propto \widetilde{\phi}_2 + \alpha \widetilde{\phi}_1$

Quasi linear dependencies

If basis functions $\widetilde{\phi}_1$ and $\widetilde{\phi}_2$ are very similar, then the vector $\|\mathbf{w}\| = \sqrt{2}$ corresponding to $w^h = \widetilde{\phi}_1 - \widetilde{\phi}_2$ yields $\|\mathbf{DADw}\| \ll 1$

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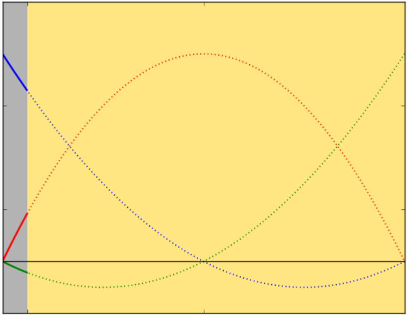
Quasi linear dependencies on nonsmooth bases

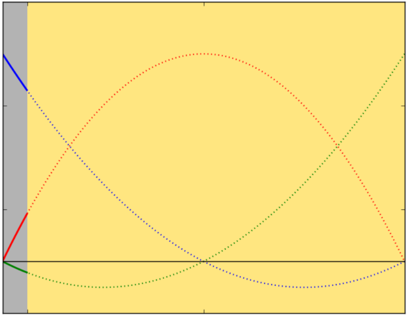


Original basis $\boldsymbol{\Phi}$

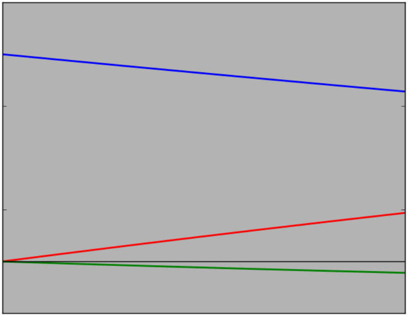
Quasi linear dependencies on nonsmooth bases

Quasi linear dependencies are a frequent phenomenon on high order bases with low regularity!

Quasi linear dependencies on nonsmooth bases



Original basis $\mathbf{\Phi}$

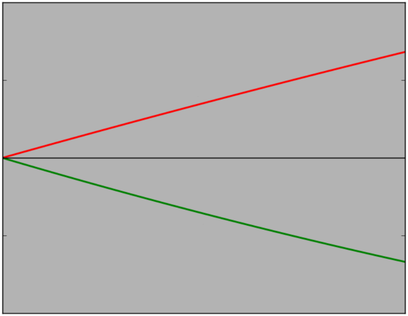


Restricted basis $\mathbf{\Phi}$

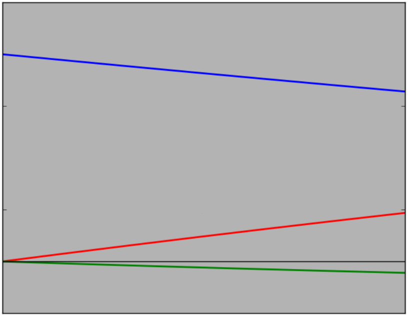
Quasi linear dependencies on nonsmooth bases

Quasi linear dependencies are a frequent phenomenon on high order bases with low regularity!

Quasi linear dependencies on nonsmooth bases



Scaled basis $\widetilde{\mathbf{\Phi}} = \mathbf{D}\mathbf{\Phi}$



Restricted basis $\mathbf{\Phi}$

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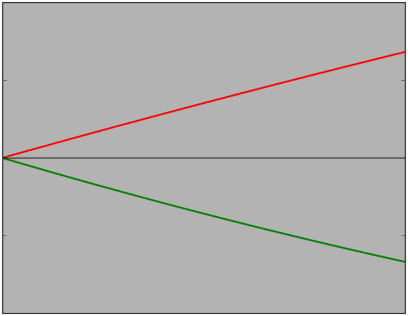
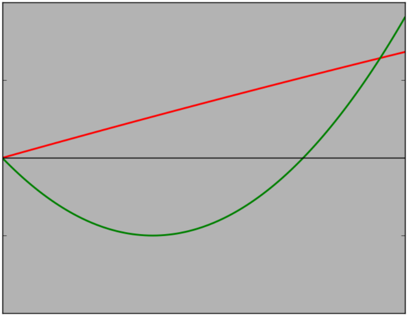


Scaled basis $\widetilde{\mathbf{\Phi}} = \mathbf{D\Phi}$



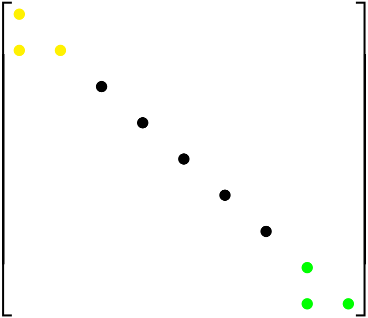
Orthonormalized basis $\overline{\mathbf{\Phi}} = \mathbf{S\Phi}$

Quasi linear dependencies on nonsmooth bases

Quasi linear dependencies are a frequent phenomenon on high order bases with low regularity!

Generalization for non-SPD problems

$\mathbf{S} =$

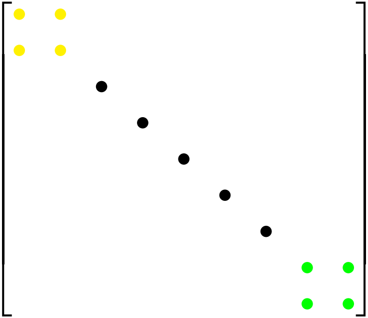


Interpretation

$\mathbf{S}^T\mathbf{S}$ is equal to the inverse of $\mathcal{R}(\mathbf{A})$, which is the restriction of matrix $\mathbf{A}$ to its diagonal and the blocks of quasi linear dependent functions

Generalization for non-SPD problems

$\mathbf{S}^T\mathbf{S} =$



Interpretation

$\mathbf{S}^T\mathbf{S}$ is equal to the inverse of $\mathcal{R}(\mathbf{A})$, which is the restriction of matrix $\mathbf{A}$ to its diagonal and the blocks of quasi linear dependent functions

Generalization for non-SPD problems

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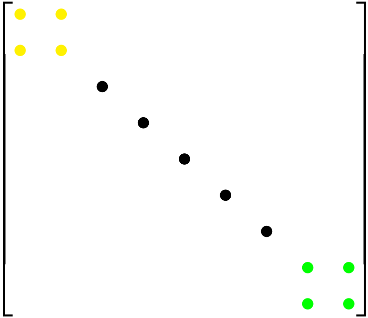


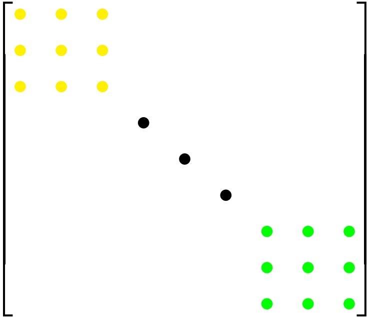
$= \mathcal{R}(\mathbf{A})^{-1}$

Interpretation

$\mathbf{S}^T\mathbf{S}$ is equal to the inverse of $\mathcal{R}(\mathbf{A})$, which is the restriction of matrix $\mathbf{A}$ to its diagonal and the blocks of quasi linear dependent functions

Generalization for non-SPD problems

$\mathbf{S}^T\mathbf{S} =$



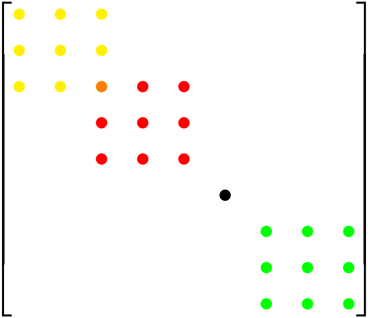
$= \mathcal{R}(\mathbf{A})^{-1}$

Interpretation

$\mathbf{S}^T\mathbf{S}$ is equal to the inverse of $\mathcal{R}(\mathbf{A})$, which is the restriction of matrix $\mathbf{A}$ to its diagonal and the blocks of quasi linear dependent functions

Generalization for non-SPD problems

$$\mathbf{S}^T\mathbf{S} = [\ldots] \neq \mathcal{R}(\mathbf{A})^{-1}$$

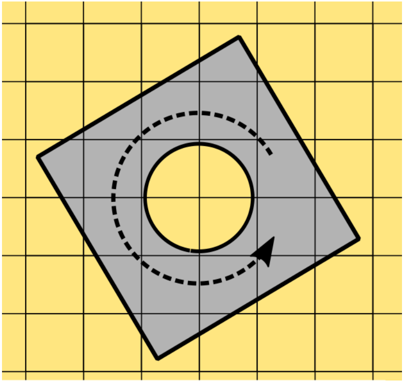


Interpretation

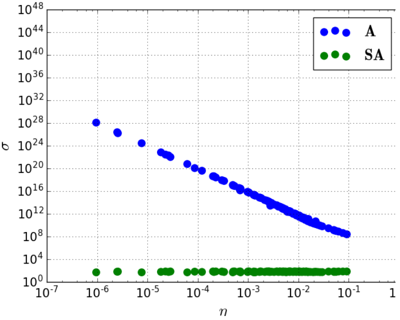
Additive-Schwarz preconditioning

Results for flow problems

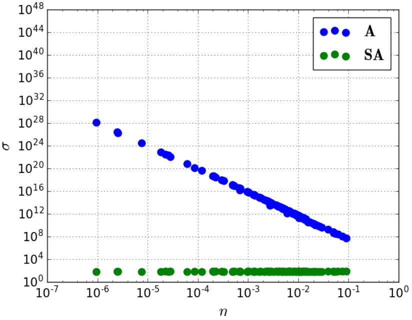
Domain:



$p = 2$



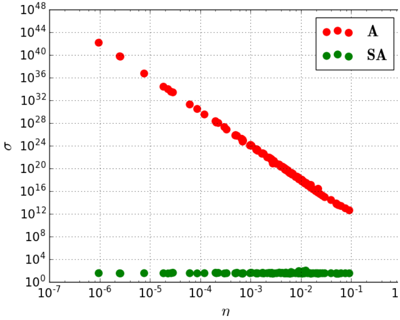
Stokes



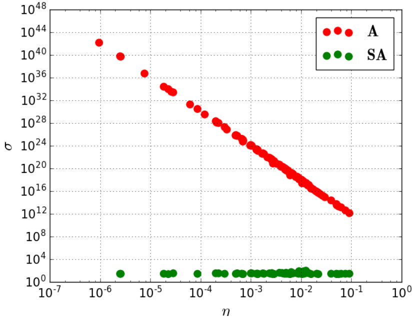
Navier-Stokes

Results for flow problems

$p = 3$

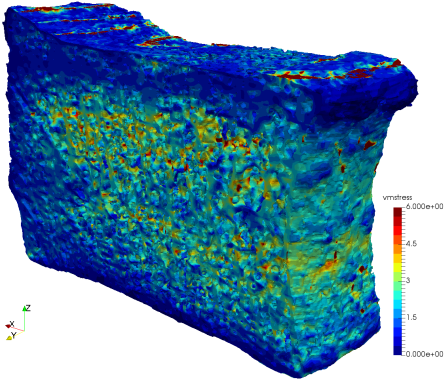


Stokes



Navier-Stokes

Results for elasticity problems

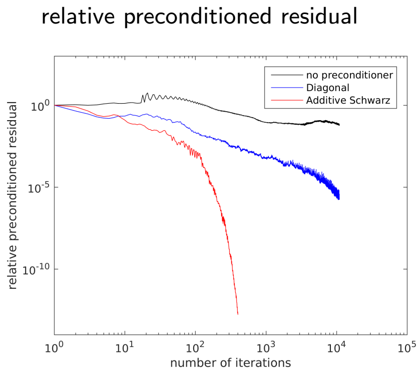


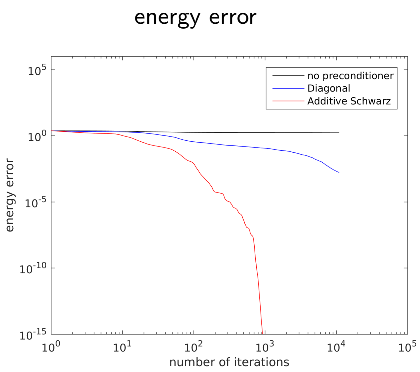
CT-scan of human vertebra

John Jomo
Collaboration with: Stefan Kollmannsberger
Ernst Rank

Technische Universität München

Results for elasticity problems





John Jomo
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Technische Universität München

Conclusion

Summary

- Introduction to immersed finite elements methods
- Conditioning analysis
- Effective tailored preconditioner

Future work (in immersed methods)

- Preconditioning
  - Combinations with other (multigrid) preconditioners
  - Parallel and meshless implementations
- Explicit dynamics
- Compatible (divergence free) discretizations
- Multiphase flows

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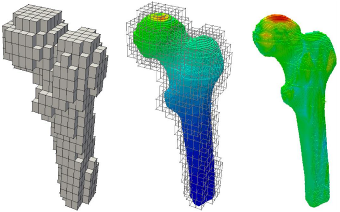
Advanced School on

Immersed Methods



06-09 November 2017

Eindhoven University of Technology

Topics

Fundamental modelling aspects
Boundary and coupling conditions
Numerical integration techniques
Ghost penalty
Conditioning and solution methods
Image-based modelling
Application in fluid and solid mechanics
Application in isogeometric analysis
Application in topology optimization

Registration before October 31st 2017
Website: www.tue.nl/emiworkshop
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Umeå University
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Technical University of Munich
Martin Ruess
University of Glasgow
Ole Sigmund
Technical University of Denmark
Clemens Verhoosel
Eindhoven University of Technology

DTU Technical University of Denmark
EPFL École Polytechnique Fédérale de Lausanne
TU/e Technische Universiteit Eindhoven University of Technology
TUM Technische Universität München